# Pricing access to the Internet with partial information

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**Abstract**

We shall consider two competition problems between service providers with asymmetric information. The utility of each one of them depends on the demand it gets and in its price. The demand itself is also a function of the prices of the providers. In both problems there is one provider (player 1) that has more information than the other (player 2) on the demand function. The more informed provider plays first, and then the second observes the move of the first provider and chooses accordingly its own action: it determines its price per unit demand. In the first problem that we consider, the first provider does not control its price (it has a fixed price known to the other provider which does not depend on the information that is unknown to provider 2). Before player 2 takes its action it receives a signal (or a recommendation) from the more informed player, i.e. from provider 1. The pure actions of provider 1 are thus the possible choices of what signal to send. The second problem that we consider is the same as the first one except that the actions of provider 1 is to choose its price. Since player 2 observes the choice of price of player 1 before it takes its own pricing decision, we can consider the choice of price by player 1 has also a role of signalling. We reduce each one of the problem to an equivalent four by four matrix game.

## 1 Introduction and Model

This note uses Bayesian game theory in order to study pricing issues in competitive environment. We model two problems related to pricing under the assumption that only one mobile fully knows the demand function. In the first part we study the signalling problem in which the informed player can signal information to the other one and has to decide on a signalling rule such that, given an optimal reaction of the other player reacts to the signal, the utility of the informed player would be maximized. In a second sceario that we study, each of the two players determine their prices. Yet the order of the decisions is such that the player that is not informed of the demand function that is used, determines its own price after observing the choice of the first player. We formulate both games as Bayesian games and show how to transform them into equivalent matrix games.

Our starting point is to define two service providers. The demand $d_i$ that provider $i$ receives is a function of both its own price as well as of its competitor. We thus write it as $d_i(p_1, p_2)$. We assume that the utility for provider $i$ is given by

$$U^i = d_i p_i, \quad i = 1, 2$$

Each provider wishes to maximize the *expectation* of its own utility.

## 2 The Signalling Game

Let us first consider the following scenario, which is common knowledge for both providers (which we call players).

- The price $p_1$ of player 1 is fixed and known.
- Player 2 can choose one of two possible levels of pricing: $\zeta_l$ and $\zeta_h$.
- The demand $d_i$ for each provider is a function of the prices of both providers. The demand function of player 2 is known to both players.
- We assume that the demand of player 1 depends on some parameter $\theta$. $\theta$ is known to player 1 but not to player 2. It can take one of two values: $\theta_1$ and $\theta_2$.

The game is then played as follows.

- The value of $\theta$ is chosen by nature according to some distribution $\pi$. It equals $\theta_r$ with probability $\pi_r$.
- Player 1 observes $\theta$ and then sends a signal to player 2. It has four pure strategies for signalling: $q_1q_1$, $q_1q_2$, $q_2q_1$ and $q_2q_2$. Here, $q_iq_j$ is the strategy according to which player 1 signals $q_i$ if it observes $\theta_1$ and signals $q_j$ if it observes $\theta_2$.
- Player 2 has also four pure strategies. We write them as $\zeta_l\zeta_l$, $\zeta_l\zeta_h$, $\zeta_h\zeta_l$ and $\zeta_h\zeta_h$. Here, a strategy of the form $\zeta_i\zeta_j$ means choosing $\zeta_i$ if player 2 observes a signal $q_1$ by player 1 and choosing $\zeta_j$ otherwise.

The game that we study is known as a game with "cheap talk"; this is a signaling game where the sender incurs no cost for his signals [1, Sec. 7].

### 2.1 Transformation into a matrix game

We can now represent the game as a $4 \times 4$ matrix game.

Fix $i \in (1,2)$, $j \in (1,2)$, $m \in (l,h), n \in (l,h)$. Assume that player 1 chooses $q_iq_j$ and that player 2 chooses $\zeta_m\zeta_n$.

For $r = 1,2$, define $\xi(r,i,j)$ to be $i$ if $r = 1$ and $j$ otherwise. Player 1 sends signal $q_{\xi(r,i,j)}$ to player 2 if $\theta = \theta_r$.

Define $\phi(r,i,j,m,n)$ to be the price chosen by player 2 as a result of the signal $\xi(r,i,j)$ when it uses the strategy $\zeta_m\zeta_n$. $\phi(r,i,j,m,n)$ equals $\zeta_m$ if $\xi(r,i,j) = 1$ and is otherwise $\zeta_n$.

Then

$$U^1(q_iq_j;\zeta_m\zeta_n) = \mathbb{E}_{q_iq_j;\zeta_m\zeta_n}[p_1d_1(p_1,p_2)] \tag{1}$$

$$= \sum_{r=1,2} \pi_r \mathbb{E}_{q_iq_j;\zeta_m\zeta_n}[p_1d_1(p_1,p_2)|\theta = \theta_r] \tag{2}$$

where $\mathbb{E}_{q_iq_j;\zeta_m\zeta_n}[p_1d_1(p_1,p_2)|\theta = \theta_r] = p_1d_1(p_1,\phi(r,i,j,m,n)$.

Thus we can rewrite:

$$U^1(q_iq_j;\zeta_m\zeta_n) = p_1 \sum_{r=1,2} \pi_r d_1(p_1,\phi(r,i,j,m,n)$$

For player 2 we obtain:

$$U^2(q_iq_j;\zeta_m\zeta_n) = \mathbb{E}_{q_iq_j;\zeta_m\zeta_n}[p_2 d_2(p_1,p_2)] \tag{3}$$
$$= \sum_{r=1,2} \pi_r \cdot \phi(r,i,j,m,n) \cdot d_2(p_1, \phi(r,i,j,m,n) \tag{4}$$

### 2.2 The Linear Case

We now consider a linear demand function:

$$d_i = d_i^0 - A_{ii}p_i + A_{ji}p_j \tag{5}$$

Assume that $A$ is known but that $d_2^0$ is unknown. It can be written as a function of a parameter $\theta$, i.e., $d_2^0 = d_2^0(\theta)$.

It follows from the above considerations that the utility functions of player 1 and player 2 can be expressed as:

$$U^1(q_iq_j;\zeta_m\zeta_n) = p_1 \sum_{r=1,2} \pi_r \Big(\theta_r - A_{11}p_1 + A_{21}\phi(r,i,j,m,n)\Big) \tag{6}$$

$$U^2(q_iq_j;\zeta_m\zeta_n) = \sum_{r=1,2} \pi_r \cdot \phi(r,i,j,m,n)\Big(d_2^0(\theta_r) - A_{22}\phi(r,i,j,m,n) + A_{12}p_1\Big) \tag{7}$$

## 3 Prices as signals

Now consider the following variant of the problem. Instead of a fixed price $p_1$, the first player can choose one of two values as its own price instead of just a signle. Yet the price itself serves also as a signal that player 2 can see. $q_i$, $i = 1,2$ now stands for the price chosen by player 1.

In this case assuming that $q_i$ stands for the price chosen by player 1 and following the previous definitions, we have that $q_{\xi(r,i,j)}$ corresponds to the price chosen by player 1 if he plays strategy $q_iq_j$ and $\theta = \theta_r$. The utility functions we obtain are:

$$U^1(q_iq_j;\zeta_m\zeta_n) = \sum_{r=1,2} \pi_r \cdot q_{\xi(r,i,j)}(d_1(q_{\xi(r,i,j)}, \phi(r,i,j,m,n)))$$
$$U^2(q_iq_j;\zeta_m\zeta_n) = \sum_{r=1,2} \pi_r \cdot \phi(r,i,j,m,n)(d_2(q_{\xi(r,i,j)}, \phi(r,i,j,m,n)))$$

### The Linear Case

If we consider again the linear demand function defined in (5), the utility functions of player 1 and player 2 can be expressed as:

$$U^1(q_iq_j;\zeta_m\zeta_n) = \sum_{r=1,2} \pi_r \cdot q_{\xi(r,i,j)}(\theta_r - A_{11}q_{\xi(r,i,j)} + A_{21}\phi(r,i,j,m,n))$$

$$U^2(q_iq_j;\zeta_m\zeta_n) = \sum_{r=1,2} \pi_r \cdot \phi(r,i,j,m,n)(d_2^0(\theta_r) - A_{22}\phi(r,i,j,m,n) + A_{12}q_{\xi(r,i,j)})$$

It follows from the above considerations that $p_1 = q_{\xi(r,i,j)}$, yielding the following expressions for the utility functions of player 1 and player 2

$$U^1(q_i q_j; \zeta_m \zeta_n) = \sum_{r=1,2} \pi_r \cdot q_{\xi(r,i,j)}(\theta_r - A_{11} q_{\xi(r,i,j)} + A_{21}\phi(r,i,j,m,n)) \tag{8}$$

$$U^2(q_i q_j; \zeta_m \zeta_n) = \sum_{r=1,2} \pi_r \cdot \phi(r,i,j,m,n)(d_2^0(\theta_r) - A_{22}\phi(r,i,j,m,n) + A_{12} q_{\xi(r,i,j)}) \tag{9}$$

# 4 Discussion

In this paper, we studied a model of pricing access to the Internet and investigated the role of signalling (or recommendations) in identifying side payments (price per unit demand). We considered the case of two Internet service providers aiming at maximizing their own expected utility. Under this setting, we assume that that the amount of information made available to both ISPs on the demand function is not the same: the more informed provider plays first, and then the second observes the move of the first provider and chooses accordingly its own action.

We studied two configurations of the proposed problem. In the first configuration, one service provider (say provider 1) has a fixed price known to the other provider (say provider 2). The pure actions of provider 1 are thus the possible choices of what signal to send. In the second configuration, provider 1 is allowed to choose his prices which serves as a signal to provider 2. We then reduced each one of the configuration to an equivalent four by four matrix game and studied the equilibria

Our model is, needless to say, a mere caricature that captures certain types of interactions between ISPs. The biggest benefit is that it is tractable, as evidenced by the obtained expressions in this paper. The litmus test of its usefulness will be its ability, or otherwise, to explain some observed behavior, even if only qualitatively. Studies in this direction are ongoing. Finally, an extension of our model to account for aspects of quality of service (QoS) should be brought in to enrich the model. We hope to pursue some of these in future works.

# References

[1] D. M. Kreps and J. Sobel, “signaling”, Chapter 25 in *Handbook of Game Theory*, vol. 2, R. J. Aumann and S. Hart, Elsevier Science B.V., 1994, pp. 849-867.